Approximation of subsurface drainage discharge by De Zeeuw-Hellinga theory and its verification in heavy soils of fluvial landscape of the Cerhovice brook.

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Abstract. The subsurface drainage discharge is one of the most important indicators of the impact of the drainage systems on water management. The procedure adopted in this study is based on the application of the De Zeeuw-Hellinga theory to derive the final expression for estimation of the value of the subsurface drainage discharge.

A simple analytical approximation of the Bussinesq’s Equation was used to theoretically verify the validity of the De Zeeuw-Hellinga assumptions and to confirm the correctness of the other corresponding processes.

The formulas describing the subsurface drainage discharge were derived in the conditions of the unsteady state subsurface flow to drains. These conditions included approximately horizontal impervious layer and the Dupuit’s assumptions and Darcy’s law. No recharge to the groundwater table was realized during the drainage testing.

The applicability of the De Zeeuw-Hellinga formula and the accuracy of the analytical approximation of the subsurface drainage discharge by the Bussinesq’s Equation, were verified by the real field measurements on the heavy soils of experimental watershed area of the Research Institute for Soil and Water Conservation (RISWC) Prague-Zbraslav, Czech Republic. The same data were successfully used also for the confirmation of the accuracy of
the method for derivation of the simple analytical approximation of subsurface total drainage quantity.

It was demonstrated, that this approximation of subsurface drainage discharge by De Zeeuw-Hellinga theory could satisfactorily serve in the area of water engineering practice as an elementary tool for an immediate estimation of the values of subsurface drainage discharges from the pipe drainage systems in the saturated porous environment.

The advantage of this approximation is particularly the minimum amount of input data e.g. the basic soil hydrology data and drainage system basic design parameters. The sphere of the use of the De Zeeuw-Hellinga equations is certainly very wide. The verifications of the field test’s results and measurements reflect that the possibilities of applications and their perceived benefits to the user can be fulfilled.

Keywords: subsurface pipe drainage system, subsurface drainage discharge, De Zeeuw-Hellinga theory, Bussinesq’s Equation, unsteady drainage flow conditions
Introduction

The importance of drainage system in water management, particularly the subsurface pipe drainage system is indisputable. For large-scale territorial units in the region of South-East Asia, Africa, India or China, the existence of the subsurface pipe drainage systems is a necessity. This is particularly so when their specific natural conditions bring about permanent waterlogging (ILRI / ALTERRA / UR Wageningen 2000; Ritzema 1994).

A typical example of the importance of drainage policy in Europe are countries such as the Netherlands, Lithuania or Denmark, where the ratio of the drained area to the total area of the country is 0.72, 0.4, 0.37 respectively (RISWC 2006).

Another example of land drainage offers the UK. A large-scale land drainage scheme can be found in the Crossens catchment at Southport, situated in Lancashire, to the north of Liverpool. The Crossens catchment contains about 145 km² of low-lying land covered mostly with wet lowland peat. The necessary drainage policy is based on an upper and a lower drainage system, both systems are joined at Crossens Pumping Station at Southport, which pumps water out of the catchment to the sea (Rosolova 2006; Harris 2004)

Regarding the drainage systems, one of the many important temporary problems of the water management in the Czech Republic, is the long-lasting lack of proper maintenance of the older subsurface drainage pipe systems in the agricultural and riverine (fluvial) landscape and the estimation of its hydraulic efficiency and cost-effectiveness.

To solve these problems, the Research Institute for Soil and Water Conservation (RISWC) Prague-Zbraslav, Czech Republic, the Czech Academy of Agricultural Sciences – Water Management Section, and the Czech Committee of ICID (International Commission on Irrigation and Drainage) arranged a workshop on “Drainage of agricultural lands in the context of cultural landscape” on 3rd November 2006 in Prague (RISWC 2006a).
Shortly after, riverine landscape was flooded (Kovar and Stibinger 2008), and the excess runoff left the catchment, the soil profile remains fully saturated. The high soil water content is caused by raised water table levels. Under such conditions, the subsurface pipe drainage system as a part of the local water management structures, can optimize the groundwater regime by its hydraulic function. This ensures an effective reduction of the negative impacts of floods.

From the hydrologic point of view, it seems that the current dynamic of climate is characterized by a great number of extremes, e.g. typhoons, tropic storms, massive short-terms rainstorms and other similar natural phenomena. The negative impacts of these hydrological events very often result in severe flooding.

Some of the territorial units not only in Europe (e.g. the Netherlands), but for example in the Pacific Ocean, are situated under the sea level. Therefore they have to face the permanent impact of such conditions on the water pressure and seepage.

The floods, which struck the Czech Republic in the 1997, 2002 and 2006, positively demonstrated the importance and the reason for the drainage systems and infiltration ability of the surface layers in the landscape.

Subsurface drainage discharge, generated by the presence of the subsurface pipe drainage systems in saturated soils under the unsteady state drainage flow, comes as one of the cardinal indicators of the drainage hydrology and it has to be taken into consideration within the possible solutions of water management problems.

Should the subsurface pipe drainage system have sufficient impact on the water table and hence improve land properties for environmental protection the design of the parameters of such a system has to be based on the analysis of the subsurface drainage discharge. This discharge is the essential indicator of the impact of field drainage on water regime.
This paper reviews the procedure of the application of the De Zeeuw-Hellinga theory to obtain the final expression for estimation of the value of the subsurface drainage discharge. The methodology of the simple analytical approximation of the subsurface drainage discharge by Boussinesq’s Equation to theoretically verify the validity of the De Zeeuw-Hellinga assumptions and to confirm the accuracy of the other corresponding processes is also explained in this report.

The formulas of subsurface drainage discharge were derived with the following assumptions: the subsurface flow to drains occurred only when approximately horizontal impervious layer was present and the Dupuit’s assumptions and Darcy’s law were valid.

The correctness and applicability of the analytical approximations of the subsurface drainage discharge, which were shaped into the single equation, was verified by the real field measurements on heavy soils of the experimental watershed area of the Research Institute for Soil and Water Conservation (RISWC) Prague-Zbraslav, Czech Republic (Soukup at. al. 2000).

The data from the experimental field of the RISWC Prague-Zbraslav, which were used for the verification of the results of this research mentioned above, successfully served also for the confirmation of the accuracy of derivation of the simple analytical approximation of subsurface total drainage quantity in non-steady drainage flow (Stibinger 2003).

The use of the field data collected for this study was two fold. They were used to test and verify the principles of the derivation of various characteristics of the drainage hydrology of subsurface drainage discharges obtained by De Zeeuw-Hellinga theory and by analytical solution of the Boussinesq equation on one hand, and subsurface total drainage quantity derived by Stibinger (2003a) on the other.

The derivative equations of the subsurface drainage discharge can serve as a good elementary tool of water engineering practice for an immediate estimation of the values of subsurface drainage discharge.
drainage discharge from the subsurface pipe drainage systems in saturated soils and under the non-steady state drainage flow. Also, it can serve as a tool for the analysis of the subsurface drainage discharge hydrology.

The principal part of the procedures presented in this paper is the inseparable strand of the research of the Ministry of Agriculture Czech Republic no. NPV-MZe 2005. VRK1/TP3-DP6 (1G57040), “Methodology of Option of Optimal Variant of Flood Protection and Erosion Control in Landscape” (Kovar and Stibinger 2008a), which is solved by Czech University of Life Sciences Prague, Faculty of Environmental Sciences, Department of Land Use and Improvement.

Materials and method

Study area

To verify this numerical experiment, the measured values of the subsurface drainage discharges were used. These data were obtained from the field experimental area of RISWC, Prague-Zbraslav. An area of approximately 40 ha of drained heavy clay soils located in the experimental watershed was selected for the research purpose of the evaluation of the subsurface drainage policy impact.

The RISWC, Prague-Zbraslav experimental field forms a part of the watershed area of riverine landscape of the Cerhovice Brook. The entire watershed area is situated in the Central Czech Upland with altitude 350 – 500 m AOD and with the long-term annual average precipitation of 560 mm. All soil layers at this location have low permeability and the depth to the impervious barrier is approximately 0.9 m bellow the soil surface. The parent rock is formed from shale.

The part of the experimental field area used for the verification is drained by the subsurface horizontal parallel systematic drain system with the drain spacing $L \ (m) = 11$, average of
drain depth $h_d$ (m) = 0.75, and diameter of the lateral drain $r_0$ (m) = 0.06. This drainage system represents the typical example of the shallow subsurface drainage system of heavy soils with its low hydraulic saturated conductivity. During the process of drainage no recharge to the water table was monitored.

The drainage discharge data, which were used for verification, were chosen from June 2000 through to July 2001. The start of the time series of the daily measured values of the subsurface drainage discharges was May 7, 2001, and the end of the non steady-state drainage processing was around May 29, 2001, when the subsurface drainage rate dropped under a measurable value.

De Zeeuw-Hellinga theory

The derivation of the subsurface drainage discharge $q$ (M.T$^{-1}$) in non-steady state drainage flow was performed by means of the De Zeeuw-Hellinga theory (De Zeeuw and Hellinga 1958; Štibinger 2006). The period with non-uniform distribution of drainage recharge $R$ (M.T$^{-1}$) was divided into time intervals of equal lengths (minute, day, week, month). Then the subsurface drainage discharge $q$ (M.T$^{-1}$) can be calculated gradually, step by step, or interval by interval, for every time interval. Symbol M (respective T) represents unit of length (respective time unit).

De Zeeuw and Hellinga found out (Ritzema 1994a) that if the drainage recharge $R$ (M) in each time interval (T) is assumed to be constant, the change in the drainage discharge is directly proportional to the excess drainage recharge $(R - q)$ (M) in this time interval.

The constant of proportionality is De Zeeuw-Hellinga drainage intensity factor $a(T^{-1}) = \pi^2 KH / (P_d L^2)$, which depends on the parameters of pipe drainage system, on the hydraulic properties of the soil environment, on the position of the water table level and on the position of the impervious layer (Dieleman Trafford, 1976).
a = De Zeeuw-Hellinga drainage intensity factor (T⁻¹), K = hydraulic saturated conductivity of the drained soil environment (M.T⁻¹), H = average depth of aquifer (M), P_d = drainable pore space of the drained soil (-), L = drain spacing (M). In this case, drainage intensity factor a (T⁻¹) also expresses the hydraulic efficiency of the subsurface pipe drainage system.

The initial equation describing the De Zeeuw-Hellinga theory explained above can be expressed as:

$$\frac{\partial q}{\partial t} = a(R - q)$$  \hspace{1cm} (1)

The De Zeeuw-Hellinga’s expression in the form of equation (1) actually constitutes an ordinary differential equation where the drainage factors a (T⁻¹) and R (M) are the constants and q (M) is the unknown function of variable t. The equations of this type can be solved by separation of variables. The final form of the De Zeeuw-Hellinga drainage theory is achieved:

$$q(t) = q(t - dt)e^{-adt} + R(1 - e^{-adt})$$ \hspace{1cm} (2)

or

$$q_t = q_{t-1}e^{-adt} + R(1 - e^{-adt})$$ \hspace{1cm} (3)

Detailed description of the analysis and solution of the De Zeeuw-Hellinga drainage theory presents Stibinger (2006a). If no recharge of the water table is recorded during the water table recession via the subsurface pipe drainage system (e.g. following the rainfalls, irrigations, heavy rains or floods), then R = 0. Equation (11) can be written as a formula to approximate subsurface drainage discharge:

$$q_t = q_{t-1}e^{-adt}$$ \hspace{1cm} (4)

The first term $$q_{t-1}$$ of equation (4) has the dimension of drainage rate, drainage intensity (M.T⁻¹) and the second term $$e^{-adt}$$ is dimensionless.
With respect to the constant time interval $dt$, the initial value of the subsurface drainage discharge at the beginning of the drainage process, will be approximated from the steady state conditions. For example H. Ritzema (1994b) used in this sense Hooghoudt’s Equation.

Using the calculations formed in equation (4) and with the knowledge of the basic subsurface drainage system parameters and soil hydrology characteristics ($K$, $P$, $H$), it is possible to evaluate on step-by-step basis the subsurface drainage discharge $q_t$ (M.T$^{-1}$) in any time interval $dt$ (practically in certain time $t > 0$).

The De Zeeuw-Hellinga’s method for the calculation of the subsurface drainage discharge $q_t$ (M.T$^{-1}$) is very useful and has a wide range of use. For example, if any recharge $R$ (M.T$^{-1}$) in any time interval $dt$ is present, the subsurface drainage discharge $q(t)$ (M.T$^{-1}$) will be estimated by equation (3). Similar procedures, i.e. by the method of superposition, were presented by Kraijenhoff (1958) and Maasland (1959). Those methods were simplified by Dieleman and Trafford (1976a) for engineering drainage practice.

Practical example of the application of the De Zeeuw-Hellinga’s theory to approximate landfill drainage discharge showed Stibinger (2006b).

**Analytical approximation**

For the saturated non-steady state one-dimensional horizontal flow $q(x)$ according to the Dupuit’s assumptions, Darcy’s equation can be written as:

$$q(x) = -h(x,t)K \frac{\partial h(x,t)}{\partial x}$$

(5)

where $q(x)$ (M$^2$.T$^{-1}$) is the intensity of the saturated non-steady state one-dimensional horizontal flow, $h(x,t)$ (M) is the head of the free water table level in the soil porous environment in an arbitrary horizontal distance $x$ (M) from the origin at an arbitrary time $t$ (T) $> 0$.  

9
The change in water storage per unit surface area at an infinitely small period of time is described by the equation of continuity:

$$-\frac{\partial q(x)}{\partial x} = P_d \frac{\partial h(x,t)}{\partial t} \tag{6}$$

By substitution equation (5) with equation (6), a non-linear, partial differential equation of the second-order will be obtained:

$$\frac{\partial (h(x,t)K[\partial h(x,t)/\partial x])}{\partial x} = P_d \frac{\partial h(x,t)}{\partial t} \tag{7}$$

The non-linearity demonstrates the first part of the equation (7): $\frac{\partial (h(x,t)K[\partial h(x,t)/\partial x])}{\partial x}$. By the approximation $h(x,t) (M) = H (M)$ constant, in the first part of this equation, where $H (M) = constant$ and represents the average depth of the aquifer, the equation (7) can be formed as:

$$HK \frac{\partial^2 h(x,t)}{\partial x^2} = P_d \frac{\partial h(x,t)}{\partial t} \tag{8}$$

The equation (7) and (8) is also known as the Boussinesq’s Equation (Boussinesq 1904) and serves as a very good tool not only for description of the non steady-state groundwater flow in a general form, but also for analysis of subsurface drainage processes (Dumm 1954; Glover 1964; Dieleman and Trafford 1976c; Sagar a Preller 1980; Ritzema 1994c; Stibinger 2003b).

In this case it is assumed that no recharge to the groundwater table occurs. The unsteady-state saturated groundwater flow to the subsurface pipe drainage system, without any recharge to the water table, is accurately described by these equations.

The linearization of the equation (7) expressed by the equation (8) applies mainly in the case of deep impervious barriers. It assumes that where $H (M)$ converges to Hooghoudt’s equivalent depth $l’ (M)$, the height of the water table above the level of drain can be neglected (Ritzema 1994d).

In following case, the average depth of the aquifer $H (M)$ was approached as
$H(M) = l' + \frac{h_0}{4}$, where $h_0(M)$ is the initial water table level (M) at time $t = 0$. The simplified analytical solution of the linearized Boussinesq’s equation (8) can be presented as:

$$h(x, t) = \frac{4h_0}{\pi} e^{-\alpha t} \sin\left(\frac{\pi x}{L}\right)$$

which is the starting formula for analytical approximation of the subsurface total drainage quantity $Q(t)$ (M). Stibinger (2003c) derived this parameter into the equation:

$$Q(t) = h_0 P\left(1 - \frac{8}{\pi^2} e^{-\alpha t}\right)$$

Finally, by the differentiation of the right part of equation (10) $h_0 P\left(1 - \frac{8}{\pi^2} e^{-\alpha t}\right)$ in time $t$ the final expression of the subsurface drainage discharge $q(t)$ (M.T$^{-1}$) at the any time $t > 0$ is obtained in the form of:

$$q(t) = (8h_0 P, a / \pi^2).e^{-\alpha t}$$

Equation (11) express the value of the subsurface drainage discharge $q(t)$ (M.T$^{-1}$).

Similarly as in the case of equation (4), the first term $(8h_0 P, a / \pi^2)$ of the equation (11) has the dimension of drainage rate, drainage intensity (M.T$^{-1}$) and the second term $e^{-\alpha t}$ is dimensionless. Alongside, the equation is, of course, identical with the second term of equation (4).

This way the validity of the De Zeeuw-Hellinga theory and the accuracy of the method of the derivation was theoretically verified. This leads to the derivation of the final form of the equation (4).

The way of the analytical solution of the subsurface drainage discharge $q(t)$ (M.T$^{-1}$), which is in the final form described by equation (11), is exploitable enough. This process permits to approximate the subsurface total drainage quantity $Q(t)$ (M) in any time $t > 0$, in this case by the equation (10).
From the Darcy’s Law Ritzema (1994e) derived subsurface drainage discharge as:

\[ q(t) = h_0 \left( \frac{8KH}{L^2} \right) e^{-at} \]  

(12)

By substituting \( a(T^{-1}) = \frac{\pi^2}{P_d L^2} \) it is obvious, that the first term \( h_0 \left( \frac{8KH}{L^2} \right) \) of equation (19) is identical with the first term \( 8h_0 P.a / \pi^2 \) of equation (11). The procedure explained above affirms and supports the correctness of the method of the analytical solution of the Boussinesq Equation, leading to the final form of the equation (11).

**Results and discussion**

Measured and calculated values

The accuracy of the De Zeeuw-Hellinga’s model application represented by the equation (4) and the validity of the use of the analytical solution represented by the equation (11) were verified by real measured values of the subsurface drainage discharges. These were obtained from the field experimental area of the Research Institute for Soil and Water Conservation (RISWC) Prague-Zbraslav, Czech Republic (Soukup et al. 2000b).

The calculated values of the subsurface drainage discharges obtained by the application of equation (4) and equation (11) were compared with the daily measured data from the RISWC Prague-Zbraslav experimental watershed mentioned above.

The area of 40 hectare of the heavy soils with low hydraulic conductivity drained by systematic subsurface pipe drainage system was selected and established during years 1976/77 by the RISWC Prague-Zbraslav. The main purpose was to estimate the impact of drainage policy and particularly the force of the subsurface pipe drainage system on the water regime in agriculture landscape.

This part of drainage represents the typical example of the shallow subsurface drainage system (see Figure 1), which comprises the drain spacing \( L \) (m) = 11, average of the drain depth \( h_d \) (m) = 0.75, and diameter of the lateral drain \( r_0 \) (m) = 0.06 (Stibinger 2003d).
The soil hydrology characteristics of the drained soil layers were measured in the terrain and verified in the laboratory using the “undisturbed” core samples. Approximation of the values of the hydraulic saturated conductivity was executed by the principles and applications of the single auger-hole method with Ernst’s evaluation (Van Beers 1970), partially by an inversed single auger-hole method (Ritzema 1994f) and by double-ring infiltration method (Kutílek and Nielsen 1994). The drainable pore space (effective porosity) was approximated from the soil water retention curves (SWRC) using van Genuchten’s theory (van Genuchten and Nielsen 1985) with equivalent pore radius distribution.

Homogenous isotropic soil environment can be presented by the one value of hydraulic saturated conductivity \( K \) (m.day\(^{-1}\)) = 0.075, by drainable pore space (effective porosity) \( P_d \) (-) = 0.015, by the thickness of the low permeable soil profile 0.90 (m) and by the initial water table level \( h_0 \) (m) = 0.50.

The initial drainage and hydraulic calculations show the value of \( l' \) (m) = 0.15 (lateral drains are situated very close to an impervious layer), the value of \( H \) (m) = \( l' + \frac{h_0}{4} \) = 0.275 and indicate a value of the drainage intensity factor \( a \) (day\(^{-1}\)) = \( \frac{\pi^2 \cdot K \cdot H}{L^2 \cdot P_d} \) = 0.112.

The groundwater table levels above the drain pipes are periodically measured by the piezometers. During the water table recession (subsurface drainage processing) no precipitation, heavy rains, irrigation or floods were monitored to the groundwater table level. That means that the process of the subsurface saturated unsteady-state flow to the drains was not shaped by any recharge to the groundwater table.

The period from which the time series of the subsurface drainage discharge for verification were selected, started at the beginning of May 2001 and proceeded to the end of the month, when the unsteady state drainage process was terminated. The real measured daily values of the subsurface drainage discharge (mm.day\(^{-1}\)), from the selected period specified above, are shown in the second column of Table 1.
The results of calculations of the daily values of the subsurface drainage discharge (mm.day$^{-1}$) obtained by equation (4), based on the De Zeeuw-Hellinga drainage theory, are shown in the third column of Table 1.

Daily values of the subsurface drainage discharge (mm.day$^{-1}$) calculated according to equation (11) are shown in the last column in Table 1. These come from the simplified analytical solution of the linearized Boussinesq’s equation.

Time series of all the measured and calculated data are presented graphically in Figure 2.

**Discussion**

The daily values of the subsurface drainage discharge measured at the RISWC Prague experimental field, the values of the subsurface drainage discharge calculated by De Zeeuw-Hellinga model (4) and the values of the subsurface drainage discharge calculated by equation (11) were compared (Table 1 and Figure 2).

Graph in Figure 1 shows that the trend of the curves of the measured and calculated values is identical, although certain differences are apparent.

The course of the time series of the tested values is monotone, exponential, evidently decreasing and corresponds with the real drainage processes very well.

As can be seen in Figure 2, it appears that especially at the beginning of the tested period the values of the subsurface drainage discharge obtained by the De Zeeuw-Hellinga model approximate the process far better then the values of the subsurface drainage discharge calculated according to the equation (11).

Approximately after the three and half days of the process, the values obtained by the De Zeeuw-Hellinga theory are evidently higher then the real values, while the values estimated by equation (11) are clearly smaller (see Figure 2).
Particularly during the first days of the drainage process, the deviations (errors) of the values of the subsurface drainage discharge calculated by the equation (11) from the real measured values are rather high: The first day it is 35.2 %, in the second day 25.2 % and the third day 14.7 (see Table 2).

Dieleman and Trafford (1976c) offer an explanation of this discrepancy. They discovered that the validity of the equation (11) is defined from the certain point of time $T_p$, where the relation between subsurface drainage discharge and the decreasing of the water table level is constant. The value of the $T_p$ can be estimated from the expression $T_p$ (days) = $0.4 \div a$ (days$^{-1}$).

In this case $T_p = 0.4 \div 0.112 = 3.57$ days, which means that the equation (18) should be used just from the time 3.6 days. This way the more significant errors of the values of the subsurface drainage discharge, calculated by the equation (11) in the first three days of this drainage process are explained.

According to Mls (1984), for the saturated non-steady state groundwater flow with the use of the Dupuit’s assumptions and Darcy’s Law, the following theoretical applies: At the border (for example bank or the drain pipe) and for time on-coming (converging) to zero, the relatively extremely high values of the velocities, drainage discharges, outflows are generated. The graph in Figure 3 clearly shows, that the course of the absolute magnitude of the differences generated by the use of the equation (11) (absolute magnitude from the daily values of the measured drainage discharge minus $q(t)$ calculated by equation /18/) is monotone, slightly decreasing. Differences are directly proportional to the values of the subsurface drainage discharge. The smaller the value of the subsurface drainage discharge, the smaller the difference (error). From the time $t$ (day) = 3.57 = $T_p$ the absolute magnitude of the differences varies between 0.1 (mm.day$^{-1}$) and 0.03 (mm.day$^{-1}$), which means between 10.5 % and 3.1 %.
The time series of the absolute magnitude of the differences between the daily values of the measured subsurface drainage discharge and the values of \( q_t \) calculated by De Zeeuw-Hellinga theory also views the graph in Figure 3. During the entire observed period the course of the differences is fluctuating, from the fourth day slightly decreasing, and the absolute magnitude of the differences varies between 0.11 (mm.day\(^{-1}\)) and 0.02 (mm.day\(^{-1}\)), i.e. between 11.5 % and 2.1 % (see Table 2 and Figure 2).

From the fourth day of the drainage process, the course of the differences from De Zeeuw-Hellinga applications is almost identical with the course of the differences from calculation by equation (11).

It is perceived that the final equation (4), derived from the base of the De Zeeuw-Hellinga theory, can serve as a good tool for approximation of the subsurface drainage discharge in the whole course of the drainage process, while equation (11), which is more known and used, is suitable from the certain time of the drainage process. In this case it is from the time \( t \text{ (day)} = 3.6 \text{ day} \).

**Conclusions**

It is indisputable that the subsurface drainage discharge falls within the most important indicators of the subsurface pipe drainage system and all drainage hydrology. The correct estimation of this characteristic plays key role in drainage policy and can serve not only for the evaluation of the impact of the existing subsurface drainage system, but also for the design of parameters of a new one.

The accuracy of the De Zeeuw-Hellinga’s method in the general form was theoretically confirmed by the results of the analytical solution of the Boussinesq’s Equation.

By the real daily measured values of the subsurface drainage discharges from the experimental field of the RISWC Prague it was demonstrated that the equation (4), as a result
of the De Zeeuw-Hellinga theory, is an acceptable tool to approximate the subsurface drainage discharge under the unsteady state drainage flow in heavy soils.

The applicability of the De Zeeuw-Hellinga's assumptions in the high permeable porous environment was controlled in the field of landfill hydrology by the analysis of the measured data of the landfill leachate rate from the internal landfill drainage system at the Osečna Landfill in the Czech Republic (Stibinger 2006b).

A great privilege of the De Zeeuw-Hellinga equation (4), compare to equation (11), consists in the possibility of its use also and especially at the very beginning of the drainage process. The equation (11) or other formulas derived on a base of the analytical solution of the Boussinesq Equation do not offer this possibility. Such equations can be used only from a certain time $t$ (Dieleman and Trafford 1976d).

This procedure (policy), of course, is not usable in the case of the application of the equation (11) or in the case of the use of the other formulas derived on the basis of the analytical solution of the Boussinesq Equation, which can be used just only from the certain time $t$ (Dieleman and Trafford 1976e).

Under the umbrella of the Ministry of Agriculture Czech Republic, the Department of Land Use and Improvement, Faculty of Environmental Sciences, Czech University of Life Sciences Prague was commissioned to start a research no. NPV-MZe 2005. VRK1/TP3-DP6 (1G57040), named “Methodology of Option of Optimal Variant of Flood Protection and Erosion Control in Landscape” (Kovar and Stibinger 2008b).

The practical engineering applications of all forms of the De Zeeuw-Hellinga assumptions with other procedures are used just in this project. The aim is to estimate the values of the subsurface drainage discharge in the concrete area of interest in the landscape. Despite the necessary use of models such as DRAINMOD (Skaggs 1999), SWAP (Dam 2000), MODFLOW in the complex cases of drainage policy (unsaturated zone, cracked soils,
transient drainage processes), the application of the algorithms and the calculators in the style of equation (3) and (4) has its own not negligible advantages.

De Zeeuw-Hellinga drainage model approximation yields slightly higher values of subsurface drainage discharges than the actual data. This makes the application of its results useful in the design of effective subsurface pipe drainage systems. It can also serve as a simple and suitable engineering tool for immediate estimation of the value of subsurface drainage discharge, which requires only minimum input information (e.g. the basic soil hydrology data and drainage system basic design parameters).

Last but not least the results of the De Zeeuw-Hellinga theory in the single final form, as represented by equation (3) and (4), allow their use (can be used) in an inversion situation. This means that the equations can be used not only for the estimation of the impact of subsurface field drainage systems on the water regime in landscape, but also for verification of the design parameters of the existing subsurface drainage systems, or to determine a new one.

The sphere of the use of the De Zeeuw-Hellinga equations is very wide, all the verifications of the field test results and measurements reflect that the possibilities of applications and their benefits to the user, as mentioned above, can be fulfilled.

The verification of the field test results and measurements reflects, that the possibilities of application and their benefits user, mentioned above, can be fulfilled.

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